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The effects of colored quark entropy on the bag pressure

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Abstract

We study the effects of the ground state entropy of colored quarks upon the bag pressure at low temperatures. The vacuum expectation values of the quark and gluon fields are used to express the interactions in QCD ground state in the limit of low temperatures and chemical potentials. Apparently, the inclusion of this entropy in the equation of state provides the hadron constituents with an additional heat which causes a decrease in the effective latent heat inside the hadronic bag and consequently decreases the non-perturbative bag pressure. We have considered two types of baryonic bags, Δ and Ω^-. In both cases we have found that the bag pressure decreases with the temperature. On the other hand, when the colored quark ground state entropy is not considered, the bag pressure as conventionally believed remains constant for finite temperature.

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1 Introduction

The entropy is a concept which has taken on many meanings throughout the sciences. Its usual sense relates the heat changes to the likelihood of the related processes at various determined temperatures. In the limit of low temperatures Planck [1] noted that a mixture of different substances retained a finite entropy even at absolute zero. This result is quite contrary to the usual interpretation of Nernst's heat theorem, for which the entropy should vanish in the low temperature limit. It was Schrödinger [2] who pointed out a similar observation for N atoms each with a two level ground state. In this case we should take into consider besides the *thermal* entropy an additional entropy of value $N \ln 2$ with Boltzmann constant k taken to be unity. Schrödinger's result [2] can be readily attained for a system of N spin one half states. Apparently, these results are consistent with the principle of degeneracy and particle distinguishability. Consequently, it is consistent with the statistical definition of the quantum entropy using the density matrix ρ. The latter relates directly to the wavefunction. For a recent review see [3] and the references therein. This *quantum* entropy is given by the trace over the quantum states as follows:

$$S = -\ \mathrm{Tr}\ \rho \ln \rho \tag{1}$$

Although the name "Quantum Entropy" implies the construction of the density matrix ρ from the quantum states, the actual mathematical form is well rooted in the laws of classical physics [1]. The entropy of mixing of different types of ideal gases with constant particle number, volume and temperature, can be calculated in the same way as Eq. 1 by replacing ρ with x_i, which is just the proportion of each constituent type i in the total gas system. Thus the additional part of entropy (entropy of mixing) becomes $-\sum_i x_i \ln x_i$, where the sum has replaced the trace operation. This expression is clearly a constant independent of the temperature so that it must remain at absolute zero [1]. In the Lie algebra the quantum fluctuation and correspondingly the quantum entropy are given by non-zero commuting set. In general, this kind of entropy represents the uncertainties in the abundance of information, for which there is an upper bound of accuracy given by the quantum uncertainty principle [4].

In a recent work we have applied these ideas to the quark singlet ground state of the hadrons [5]. The color symmetry $SU(3)_c$ provides an entropy for

each of the colored quarks with the value $\ln 3$. We have extended this result to models at finite temperatures [6]. This entropy reflects the probability of quark mixing maneuver inside the hadrons. At vanishing temperature the confined quarks can be viewed as continuously tousled objects. In present work we investigate the contribution of the ground state entropy to the equation of state for the colored quarks using the phenomenological bag model for strong interactions [7]. We assume that inside the hadron bag all of the strong interactions at low temperatures T and small quark chemical potentials μ_q are included in the quark and gluon condensates. We will not elaborate here any further on details of the bag model. We apply it as a simplest analytical model for the QCD equation of state. In this model and in the low temperature limit the thermodynamical quantities of the quarks are given by the non-perturbative bag pressure, which is widely known as the bag constant $\mathbf{B}$. $\mathbf{B}$ gives the gain in the energy density of the confined state relative to its value in vacuum (section 3). In particular, we will look at a special model for baryons, for which the effective degrees of freedom are entirely given by the quark and gluon colors. All other couplings are taken so that the spin and flavor are not explicitly considered.

Hypothetical colored quarks have been suggested as an intermediate phase in the confinement-deconfinement transition. At high chemical potentials and low temperatures the hadronic matter has been conjectured to dissolve into degenerate fermionic quarks. This cold dense quark matter is believed to exist in the interior of compact stars. The phenomenological behavior of colored quarks at vanishing T has been discussed in [8]. These are some examples, in which this part of entropy should be taken into account.

2 Quark and gluon condensates

In the limit of low temperatures and chemical potentials the interactions in the QCD ground state are expressed in terms of the vacuum expectation values of the quark and the gluon fields. The calculation of these vacuum contributions are gotten from the operator product expansion using the QCD sum rules [9, 10], which has the local operators of dimension four yielding the main contributions to the thermodynamics [11, 12, 13]. The pure gluon vacuum expectation value is calculated [14] from the product of the field strength tensors $G^a_{\mu\nu}G^{\mu\nu}_a$ including the non-zero renormalization group $\beta(g)$-

function [11].

$$< G^2 >_0 \;=\; \frac{-\beta(g)}{2g^3} G^a_{\mu\nu} G^{\mu\nu}_a \qquad (2)$$

where the repeated indices are summed over their range of values. From here on the subscript in $< \cdots >_0$ refers to $T = 0$. This gluon condensate can be extracted [9] from the charmonium spectrum to yield a consistently estimated [7] value of about 1.95 GeV/fm^3.

For the quark condensates we consider two extreme cases for vacuum expectation values:

$$\begin{aligned} m_q < \bar{q}q >_0 &= m_{lq} < \bar{u}u + \bar{d}u >_0 \quad \text{pure light quarks} \\ &= m_s < \bar{s}s >_0 \qquad\qquad \text{pure strange quarks} \end{aligned} \qquad (3)$$

The operator for the pion decay relates the product of light quark condensate and the corresponding mass to the fixed value $-m_\pi^2 f_\pi^2$, f_π is the pion decay factor. For strange quarks we can apply a similar relation with $-m_K^2 f_K^2$. In this case f_K stands for the kaon decay. We use for light quark mass $m_q = m_u \equiv m_d = 6$ MeV and strange quark mass $m_s = 150$ MeV. With these values together with $m_\pi = 138$ MeV and $m_K = 496$ MeV [15] we find the light and strange quark condensates are -42 MeV/fm^3 and -273 MeV/fm^3, respectively. The averaged vacuum contribution to the fields of dimension four in the equation of state can be calculated from the operator product expansion [14] using

$$< \Theta^\mu_\mu >_0 \;=\; < G^2 >_0 + m_q < \bar{q}q >_0 \qquad (4)$$

As in [11] the thermal decay of the quark and gluon condensates is different. Meanwhile the quark condensate shows a thermal dependence up the order of T^8, the gluon condensate has a much slower decay. Therefore, we assumed in Eq. 4 that the two condensates are T-independent, especially in the low temperature limit. Furthermore, we remark here that both condensates have the same color singlet ground state 0^{++} often associated with the scalar glueball state [16].

As we mentioned above, we have chosen two extreme cases for the baryonic structure. If we look at the spin 3/2 structure of Δ and Ω^- as examples of these ground state structures, we get a minimal effect from the spin entanglement and flavor mixing. Singly, the quark and gluon colors are assumed

to be asymmetric. Nevertheless, we include the degeneracy factor due to the quark spins for the integration over the momenta. The usual sum over the flavors is replaced by a factor of three in the baryons. We keep the degeneracy factor due to the gluon spins since both polarizations are possible. We shall use the trace anomaly for the substitution of the above extracted values for the vacuum condensates into the equation of state. As discussed above, it is known [11, 12, 13] that the temperature has very little effect on these values at temperatures well below 100 MeV. Thus we can look at the quantum effects of the entropy on the bag pressure $\mathbf{B}$ for low temperatures. We do not look at the explicit dependence of the bag pressure on the quark chemical potential μ_q. Another reason for excluding the μ_q dependence is that the colored quark entropy is not given in terms of μ_q (Eq. 8).

3 The Equation of State

At finite temperatures and zero quark chemical potentials we choose the grand canonical partition function $\mathcal{Z}(T, V, \mu_q = 0)$ in order to write down the equation of state in terms of difference between the energy density and the pressure $\varepsilon(T) - 3p(T)$. Hereupon, we describe the expectation values for the gluon and quark condensates from the equation of state as,

$$\lim_{T\to 0} < \Theta^{\mu}_{\mu} >_T \;=\; \varepsilon(T) - 3p(T) \tag{5}$$

where the repeated indices represent the sum over the Lorentz indices. In the formulation of the bag model [7] the thermodynamics is usually included with a bag energy density $\varepsilon = +\mathbf{B}$ and a bag pressure $p = -\mathbf{B}$, which generally represents the energy density and the confining pressure of the bag against the vacuum. After applying the first law of thermodynamics which relates the entropy density to the internal energy and pressure densities

$$Ts(T) \;=\; \epsilon(T) + p(T) \tag{6}$$

we find that Eq. 5 reads

$$\left\langle \Theta^{\mu}_{\mu} \right\rangle_T \;=\; T\left[\frac{3\mathcal{S}_{q,3}(T)}{V} + s(T)\right] - 4\left[p(T) - \mathbf{B}\right] \tag{7}$$

Here we include the ground state entropy $\mathcal{S}_{q,3}$ [5, 6] and the bag pressure $\mathbf{B}$. The latter is usually assumed as independent of the parameters of the

ensemble. The l.h.s is estimated as in Eq. 4. For $T \to 0$ we get back Eq. 5, for which $< \Theta_\mu^\mu >_0 \to 4\mathbf{B}$, according to the bag model.

Since the effective degrees of freedom considered here are merely the colors of quarks and gluons, we can apply our model [6] in calculating $\mathcal{S}_{q,3}$.

$$\begin{aligned}\mathcal{S}_{q,3}(T) &= -\frac{1}{3}\left(1-e^{-m_q/T}\right)\ln\left[\frac{1}{3}\left(1-e^{-m_q/T}\right)\right] \\ &\quad -2z\left[\ln(z)\cos(\theta)-\theta\sin(\theta)\right]\end{aligned} \tag{8}$$

where the values for z and θ respectively are

$$\begin{aligned} z &= \frac{1}{3}\left(1+e^{-m_q/T} \;+\; e^{-2\,m_q/T}\right)^{1/2}, \\ \theta &= \arctan\left(\frac{\sqrt{3}\,e^{-m_q/T}}{2+e^{-m_q/T}}\right). \end{aligned} \tag{9}$$

The other thermodynamic quantities in Eq. 7 are given as follows:

$$p(T) = \frac{3}{\pi^2}\,T\int_0^\infty k^2\,dk\ln\left(1+e^{-\frac{\epsilon(k)}{T}}\right)+\frac{8\pi^2}{45}T^4 \tag{10}$$

$$s(T) = \frac{3}{\pi^2}\,\frac{1}{T}\int_0^\infty k^2\,dk\frac{\epsilon(k)}{e^{\frac{\epsilon(k)}{T}}+1}+\frac{p_q(T)}{T}+\frac{32\pi^2}{45}T^3 \tag{11}$$

where $\varepsilon(k)^2 = m_q^2 + k^2$ is the single particle energy. Equations 10 and 11 give the pressure and the entropy density inside the baryonic bag in depending on T in the usual way. In Eq. 11 the expression $p_q(T)$ means just the quark contribution to the pressure in the first term of Eq. 10. The contributions of gluons to these quantities are also taken into account, for which we considered the spin degeneracy due to the possible polarization. However, for the sake of simplicity, we have given the gluon radiation in the grand canonical partition function in an approximated form[1] which, for instance,

[1] At high temperatures the pressure of equilibrated ideal bosonic gas of gluons reads

$$p(T)_{gluon} = \frac{8}{45}\pi^2\,T^4\left[1-\frac{15\alpha_s}{4\pi}+\cdots\right]$$

where α_s is the running strong coupling constant, which depends on T. At low temperatures we can take $\alpha_s \to 0$.

appears in the second term of Eq. 10. This approximation is appropriate since T remains small compared to Λ_{QCD}.

By using the values of vacuum condensates given above, we can numerically solve Eq. 7 to get the dependence of the bag pressure $\mathbf{B}$ upon the temperature T. The universal constant of the free-space bag pressure is still debatable. A temperature dependence with the effective Lagrangian for gauge fields has been reported in [17]. Also according to the finite density QCD sum rules a large reduction in $\mathbf{B}$ is expected [18]. For the two special baryonic structures of the hadron bag model, we shall compare the constancy of the bag pressure with and without the ground state entropy $\mathcal{S}_{q,3}$ for the colored quarks. The inclusion of ground state entropy term [5, 6] given in Eq. 8 when set into the equation of state Eq. 7 is to be viewed as providing with additional heat and therefore has the effect of decreasing the thermal pressure of colored quarks and gluons. This leads to decreasing in the value of $\mathbf{B}$ needed to preserve the hadron bag's stability against the force of the outside vacuum.

4 Results and Discussion

In Fig. 1(a) we plot $\mathbf{B}$ as a function of T for the baryonic state Δ while leaving out in Eq. 7 the quantum entropy contribution [5, 6]. Here we have used the vacuum expectation value 1.91 GeV/fm^3. We notice that $\mathbf{B}$ remains constant, especially at very low temperatures. Afterwards it only slightly increases so that over 30 MeV temperature, the bag pressure only gains 0.04 MeV/fm^3. The reason for the increasing is that the thermal pressure of baryon constituents (Eq. 10) increases faster than $Ts(T)$ (Eq. 11). But at low temperature ($T < 20$ MeV) both pressure and entropy density are too small and consequently almost equally badly. Therefore, $\mathbf{B}$ clearly remains constant. Another reason for increasing $\mathbf{B}$ for $T > 20$ MeV may be the absence of the chemical potential. If the calculations were done for finite μ_q, we have to modify equations 10 and 11 and correspondingly add the positive term $3\mu_q\rho(T,\mu_q)$, where $\rho(T,\mu_q)$ is the quark number density. In this case $Ts(T)$ together with this term may be able to balance the increasing $p(T)$. Therefore, we can conclude that $\mathbf{B}$ almost remains constant for a wide range of temperatures.

In Fig. 1(b) we show the corresponding results for the baryonic state Ω,

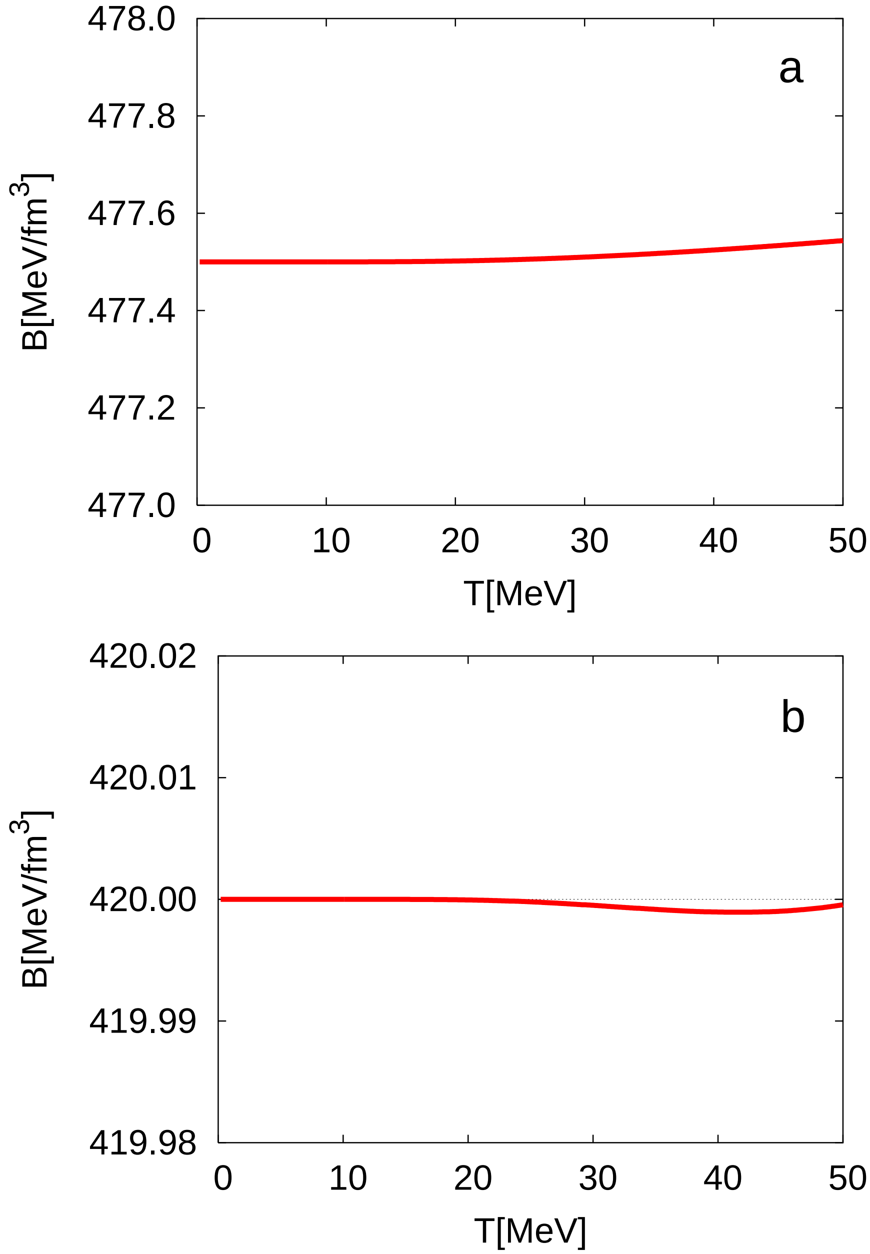

Fig. 1: The panel (a) depicts **B** as a function of T in the baryonic system with the three light quarks. Here $\mathcal{S}(T)_{q,3}$ is not included in the equation of state. The bottom panel gives the same dependence for the baryonic bag of the three strange quarks.

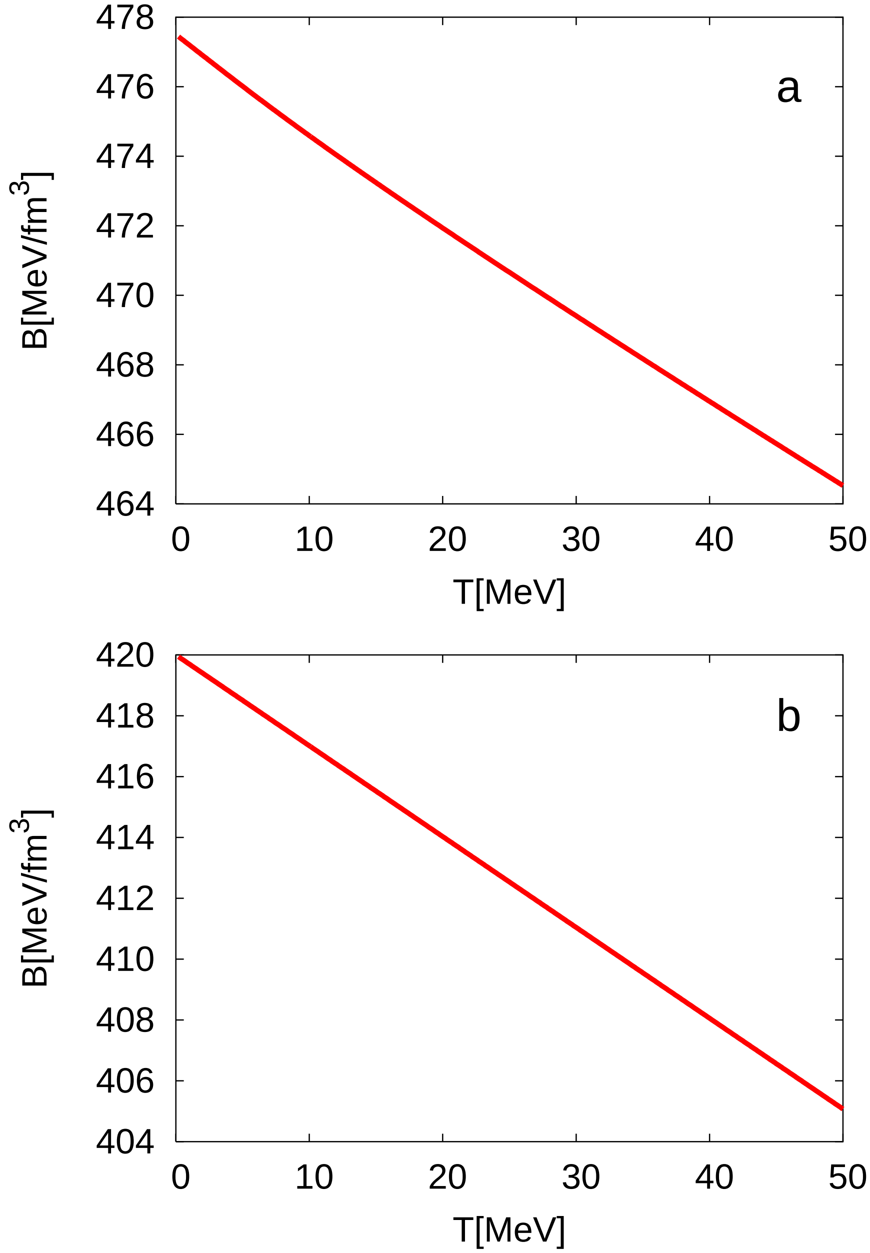

Fig. 2: The top panel shows the thermal structure of $\mathbf{B}$ for the three light quarks. Similar for the three strange quarks appears in (b). For both of them we added the ground state entropy $\mathcal{S}(T)_{q,3}$ in Eq. 7.

again without the quantum entropy contribution. Here we have used the vacuum expectation value 1.68 GeV/fm^3. In this case we note the range of T in which $\mathbf{B}$ remains absolute constant is almost the same as in the case of Δ. Beyond this range it decreases only marginally. Thereafter it starts to increase in a region not shown here. The reason for decreasing $\mathbf{B}$ is just the inverse of that in previous case. For strange quark mass, which is very much heavier than the masses of light quarks, the pressure is much smaller than $Ts(T)$. But this is expected to be altered with the temperature. For larger T we could expect that $\mathbf{B}$ will increase in a way similar to the Δ bag.

Thus we may conclude that $\mathbf{B}$ is *almost* constant in both baryonic bags, especially at $T << T_c$, i.e. at temperatures at which the quark condensate can be determined from states of low-energy: the vacuum and the low-lying hadronic resonances, like pions and kaons [19, 20].
In the high temperature region we expect that the values of $\epsilon(T) - 3p(T)$ do not vanish, for which the gluonic radiation contributions become very dominant. Thus, we expect that at vanishing temperatures $\epsilon - 3p$ approaches $4\mathbf{B}$. Therefrom, we can determine the region of T, in which the bag pressure remains relatively unchanged.

The inclusion of the quantum entropy density leads to quite different effects. This may be seen in Fig. 2. Particularly, in Ω bag there is a linear decrease of $\mathbf{B}$ with increasing T. In Δ there is a very small bend around $T \sim 20$ MeV. Afterwards, the decay gets linear too. Since the asymptotic value of the quantum entropy density is finite for very high temperatures [6], $\mathbf{B}$ is expected to continuously decrease with increasing T.

5 Conclusion and Outlook

Some situations where the ground state entropy has essential applications are shortly discussed in section 1. The hadronic matter is expected to undergo a phase transition into degenerate quarks at low temperatures and large chemical potentials. These conditions may be present in the interior of hybrid stars and large planets and could be achieved in the future heavy-ion experiments. Present investigation can by applied in understanding the matter under these extreme conditions. Moreover, the physics of colored quarks plays a prominent role in understanding the dynamics of color superconductivity. In a recent work [21] we have calculated the ground state

entropy for a colored two-quark system and compared it with the entropy arising from the excitations in the BCS model and in the Bose-Einstein condensate (BEC) for bosonic quark-pairs at low temperatures and high quark chemical potentials. On the other hand, we need to consider the entropy for colored quarks in order to be able to calculate the consequences of QCD at low temperatures.
Understanding the thermal behavior of colored quark states is very useful for different actual applications. It may well bring about a device for the further understanding of the physics behind the recent lattice results for the entropy of static quark-antiquark in heavy quark potential [22]. The entropy difference in a quark-antiquark singlet state on the lattice gives a value of $2\ln 3$ at vanishing temperature [23]. Our estimation for the quantum entropy of *one* quark as a part of colorless singlet state [5] yields the value of $\ln 3$ [22]. Taking this in mind, we can also reproduce the lattice entropy results at finite temperatures [22]. In doing it, we take this T-independent term together with string model and the pure $SU(3)$ gauge theory results. Furthermore, we believe that the investigation of the quantum subsystems at finite temperature might be useful for understanding the concept of confinement, which could exists everywhere throughout $T \in [0, \infty]$. The quark distributions inside the hadron bags reflect themselves as entanglement or - in our language - quantum entropy.

Finally, we can conclude that the presence of the ground state (quantum) entropy density arising from the $SU(3)_c$ color symmetry in the equation of state provides a strong temperature dependence for the bag pressure $\mathbf{B}$. The inclusion of this entropy in the equation of state for the two considered baryonic structures provides their constituents with an additional heating energy and therefore leads to an almost linear decline in $\mathbf{B}$ with increasing T. We have contrasted these results in both cases to the same properties without the quantum entropy. Although, there are other possible contributions to the quantum entropy arising from different physical quantities, like the spin, however in this model the effects from the color degrees of freedom are responsible for the decrease in $\mathbf{B}$.

Based on these results we plan further studies about the behavior of the structure of confined quark matter at very low temperatures and small quark chemical potentials. A further consideration of the idea that glueballs could appear as 0^{++} state in a BEC [16] seems to be a quite promising further

point. The importance of the glueball degrees of freedom for describing the hadronic phase for temperature below T_c has already been investigated [19, 20]. Also the existence of spin-color waves in the bag we would like to study further.

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